

包装技术与工艺

基于FEM的单层E型瓦楞纸箱托盘稳态数值分析

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摘要:目的 对单层E型瓦楞纸箱托盘进行稳态分析,以便能对其结构进行优化设计。方法 首先借助弹性力学理论,通过微分方程法对单层E型瓦楞纸箱托盘的临界载荷进行推导;然后通过UG(Unigraphics NX)对单层E型瓦楞纸箱托盘进行建模,利用试验测定的单层E型瓦楞纸箱托盘的工程常数,通过有限元法(FEM),在Ansys平台上对单层E型瓦楞纸箱托盘进行静态分析和屈曲分析,求解单层E型瓦楞纸箱托盘的屈曲模态和临界载荷。对理论方法推导和FEM试验验证的结果进行定性分析。结果 对比分析结果,仿真结果和理论推导的结果具有一致性,两种方法对比验证,为E型瓦楞纸箱的结构设计提供了理论参考。结论 理论方法推导和FEM对单层E型瓦楞纸箱托盘进行稳态分析,能够缩短产品设计周期,提升产品市场竞争力。

关键词: FEM; 瓦楞纸箱; 微分方程; 稳态分析

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Steady Numerical Analysis of Monolayer E-type Corrugated Pallets Based on FEM

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ABSTRACT: This project aimed to conduct steady-state analysis for the monolayer E-type corrugated pallets, in order to optimize the design of its structure. First of all, with the help of elasticity theory, by means of differential equations, the critical load of the monolayer E-type corrugated pallets was derived. After that, with the aid of UG, the finite element model of the monolayer E-type corrugated pallets was established. Using the experimentally determined engineering constants for the monolayer E-type corrugated pallets, by means of FEM method, static analysis and buckling analysis for the monolayer E-type corrugated pallets were conducted on the Ansys platform. Then, the buckling modes and critical load of the monolayer E-type corrugated pallets were solved. The theoretically derived and FEM test validated results were used for qualitative analysis. Comparison of the analysis results, the simulation results and the results of the theoretical derivation were consistent. The validated comparison of the two methods provided a theoretical basis for the structural design of the E-type corrugated boxes. The steady state analysis of the monolayer E-type corrugated pallets using theoretical derivation and FEM could shorten product design cycles and improve product competitiveness.

KEY WORDS: FEM; corrugated box; differential equation; steady state analysis

瓦楞纸板在包装行业广泛应用,不仅环保、可降解,而且拥有较高的抗压强度,能够有效保护被包装产品。目前在软管生产中存在瓦楞纸箱托盘变形的情

况,例如,对于 $\phi 35$ mm规格的软管,标准软管装箱容量为240支,由于瓦楞纸箱托盘的失稳变形,每箱软管装箱容量会增加至260支,如果这样的装箱误差漏检,

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直接影响下一步工业机器人的整箱抓取效果。目前,王小燕等^[1]通过有限元分析法,对瓦楞纸箱结构参数对承压能力的影响做了定量分析;滑广军等^[2]对C型楞瓦楞纸箱采用屈曲分析法计算瓦楞纸箱抗压强度,验证了屈曲分析方法对瓦楞纸箱性能研究的可行性。由于瓦楞纸箱本身比较薄,当瓦楞纸箱结构设计不合理失稳时,即使纸箱在承载低于强度和刚度的条件下依然可以导致瓦楞纸箱损坏^[3-8],因此,需要对瓦楞纸箱进行稳态分析。文中采用微分方程法和有限元方法,计算单层E型瓦楞纸板的屈曲临界载荷,为瓦楞纸箱供应商和包装设计者提供必要的理论数据。

1 单层E型瓦楞纸箱托盘稳态理论分析

1.1 E型瓦楞纸箱托盘失稳条件

设计薄板结构时,必须要考虑设计结构的稳定性问题。E型瓦楞纸板的失稳条件,即薄板的负载达到临界载荷。当E型瓦楞纸板的载荷均匀分布时,瓦楞纸板由稳定平衡到不稳定平衡过度的最小载荷,就是临界载荷。在不稳定平衡状态下,瓦楞纸板持续偏离平衡位置,或变弯的现象,称为构件屈曲。若要避免结构因失稳导致破坏,结构失稳的临界点和失稳条件是关键因素,所以要判断单层E型瓦楞纸箱托盘失稳的临界状态和求出屈曲临界载荷^[9-12]。

1.2 微分方程法计算瓦楞纸板的屈曲临界载荷

E型楞是国际通用瓦楞楞型,E型楞瓦楞纸箱托盘是通过单张标准尺寸矩形E型楞瓦楞纸分切、压痕后经手工折叠,然后胶带封边固定成型(为无盖箱体,即托盘)。单层E型瓦楞纸板内部为波浪结构,是正交各向异性材料。求解单层E型瓦楞纸板屈曲临界载荷时,应先把E型瓦楞纸板转化为标准的正交各向异性板,侧板楞向与托盘底面垂直,然后,按正交各向异性板屈曲临界载荷的解法求解^[13-15]。单层E型瓦楞纸板简化为正交各向异性板见图1,取单层E型瓦楞纸板的长宽分别为 a, b ,负载取为均布载荷,法向受力,单层E型瓦楞纸板的受力见图2。

依据弹性力学理论,对简化后的瓦楞纸板进行稳定性分析,根据小挠度理论,在单向受力下的瓦楞纸板屈曲控制微分方程。四边简支正交各向异性瓦楞纸薄板的屈曲控制微分方程为:

$$D_1 \frac{\partial^4 \omega}{\partial x^4} + 2D_3 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4 \omega}{\partial y^4} = N_x \frac{\partial^2 \omega}{\partial x^2} +$$

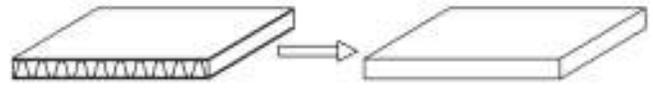


图1 单层E型瓦楞纸板简化

Fig.1 Simplified monolayer E-type corrugated board

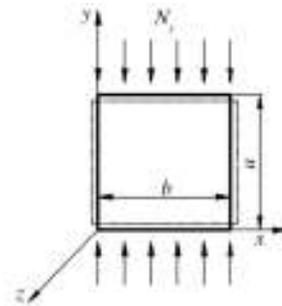


图2 单向均匀受压矩形板的屈曲

Fig.2 Buckling of rectangular plates subjected to unidirectional uniform compression

$$2N_{xy} \frac{\partial^2 \omega}{\partial x \partial y} + N_y \frac{\partial^2 \omega}{\partial y^2} \quad (1)$$

取瓦楞纸板y向为均布载荷,化简得:

$$D_1 \frac{\partial^4 \omega}{\partial x^4} + 2D_3 \frac{\partial^4 \omega}{\partial x \partial y} + D_2 \frac{\partial^4 \omega}{\partial y^4} = N_y \frac{\partial^2 \omega}{\partial y^2} \quad (2)$$

式中: D_1 和 D_2 分别为沿x和y方向的抗扭刚度; D_3 为折算刚度。

$$D_1 = \frac{E_1 h^3}{12(1 - \mu_1 \mu_2)}, \quad D_2 = \frac{E_2 h^3}{12(1 - \mu_1 \mu_2)},$$

$$D_k = \frac{G_{12} h^3}{12}; D_3 = D_1 \mu_2 + 2D_k, \text{ 且 } \frac{D_1}{D_2} = \frac{E_1}{E_2} = \frac{\mu_1}{\mu_2}.$$

屈曲临界载荷计算公式为:

$$(N_y)_{cr} = p_{cr} = \frac{\pi^2 D_2}{a^2} + 2 \frac{\pi^2 D_3}{b^2} + \frac{\pi^2 a^2 D_1}{b^4} \quad (3)$$

当 $a=b$,则:

$$(N_y)_{cr} = P_{cr} = \frac{\pi^2}{b^2} (D_1 + D_2 + 2D_3) \quad (4)$$

推导屈曲临界应力:

$$(\sigma_y)_{cr} = \sigma_{cr} = \frac{\pi^2}{b^2 h} (D_1 + D_2 + 2D_3) \quad (5)$$

2 有限元法计算单层E型瓦楞纸箱托盘的屈曲临界载荷

有限元法(FEM)是求解微分方程边值问题的一种通用的数值计算方法,不仅简便快捷、形象直观,在产品的研发阶段,设计人员就能掌握其性能参数,而且可以验证微分方程法推导的理论公式的合理性。选用有限元法研究结构或构件的平衡状态是否稳定

的问题^[16-20],通常要先进行静力分析,再进行有限元屈曲分析,有限元方法分析流程见图3。



图3 Ansys分析流程

Fig.3 The flow chart of Ansys analysis

单层E型瓦楞纸箱托盘厚度方向研究对于各向异性材料的瓦楞纸材料无意义,此次研究主要针对瓦楞纸的长宽高进行研究,同理论推导的公式进行定性分析。单层E型瓦楞纸板工程常数是经试验测定的,不同的厚度工程常数不一致,需要根据试验测定工程常数。各方向弹性模量为 $E_x=2560$ MPa, $E_y=1311$ MPa, $E_z=12.8$ MPa;各方向剪切模量分别为 $G_{xy}=955$ MPa, $G_{yz}=65$ MPa, $G_{xz}=90$ MPa;各方向泊松比分别为 $\mu_{xy}=0.24$, $\mu_{yz}=0.06$, $\mu_{xz}=0.08$ 。

在Ansys平台,通过GUI方式,对单层E型瓦楞纸箱托盘进行分析。首先,根据分析对象的特性,在Ansys单元模型库中选择常用的分析单元shell 181单元,定义材料特性为弹性,正交各向异性材料,输入试验测定的单层E型瓦楞纸板工程常数,如弹性模量、剪切模量和泊松比。然后,对导入IGS格式的UG模型进行网格划分,选择自由网格划分方式划分网格,控制网格单元尺寸为5 mm,施加单位载荷,进行静力分析。在静力分析的基础上进行单层E型瓦楞纸箱托盘的屈曲分析。Ansys分析过程无量纲,单位需自行保持一致,建模统一为N-MPa-mm单位制。

定义样本A规格为560 mm×540 mm×180 mm,样本B规格为560 mm×540 mm×120 mm,样本C规格为280 mm×270 mm×120 mm,样本D规格为280 mm×270 mm×60 mm。分别对4组样本进行六阶屈曲模态分析。根据经典理论,低阶振型有效,列出4组振型的一阶屈曲模态云图,见图4。

单层E型瓦楞纸箱托盘的第一阶屈曲模态临界载荷值计算公式^[21]为:

$$P = pfs = \sigma_{cr}s \quad (6)$$

式中: P 为单层E型瓦楞纸箱托盘屈曲时,承受的载荷(N); p 为施加给单层E型瓦楞纸箱托盘的单

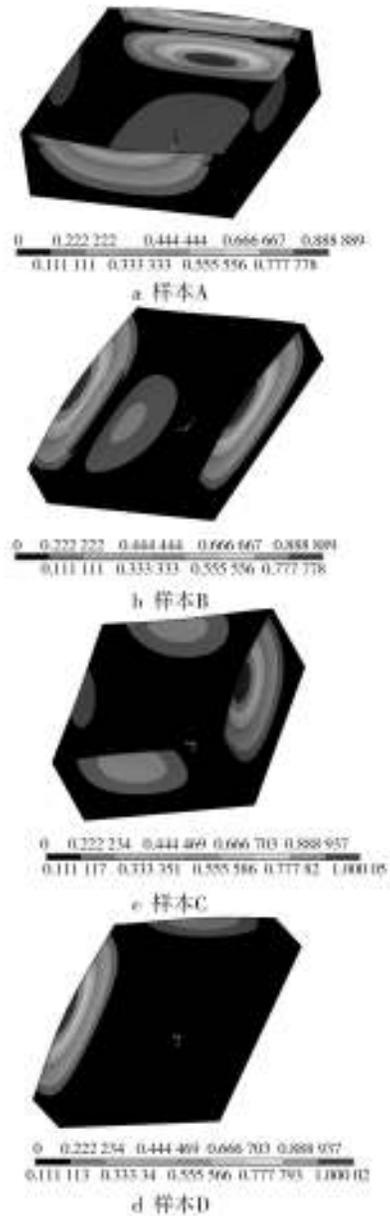


图4 4组试验对象屈曲模态云图

Fig.4 Buckling modal nephogram of the four groups of experimental objects

位载荷(N/mm²); f 为屈曲模态分析的第一阶特征值; s 为单层E型瓦楞纸箱托盘受力面积(mm²); σ_{cr} 为单层E型瓦楞纸箱托盘单位受载面积上的屈曲临界载荷(N/mm²)。利用有限元法(FEM)分析出来的单层E型瓦楞纸箱托盘的屈曲临界载荷特征值见图5。

用式(6)计算得单层E型瓦楞纸箱托盘的屈曲临界载荷,样本A规格为560 mm×540 mm×180 mm,样本B规格为560 mm×540 mm×120 mm,样本C规格为280 mm×270 mm×120 mm,样本D规格为280 mm×270 mm×60 mm的一阶屈曲模态载荷分别为 -1.78×10^5 , -2.41×10^5 , -1.22×10^5 , -2.48×10^5 N。

通过对比分析有限元分析结果为:

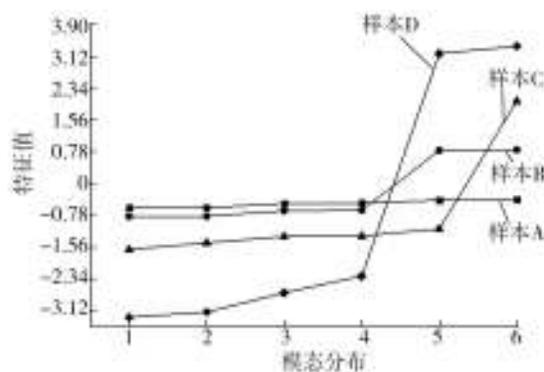


图5 4组试验对象六阶屈曲模态特征值分布

Fig.5 The sixth buckling mode eigenvalue distribution of the four groups of experimental subjects

1) 样本A与B对比,横截面相同,高度不同,当高度增大时,临界载荷 P 变小,稳定性降低。

2) 样本B与C对比,高度相同,横截面积不同,横截面积越小,承载能力 P 越大。

3) 样本C与D对比,横截面相同,高度越小,承载能力 P 越大,稳定性越好,与分析结果第一项相比,具有一致性。

4) 以上分析结果与推导的理论公式(5)长宽高参数对承载能力 P 的影响一致,即稳态影响一致。

5) 分析结果4)同图5对比也验证了经典理论,低阶振型有效。

3 结语

对于单层E型瓦楞纸箱托盘,有限元屈曲分析临界载荷能够反映纸箱的稳定性。分析了单层E型瓦楞纸箱托盘开口大小和托盘高度对E型瓦楞纸箱托盘稳定性的影响,比较了使用理论方法和有限元法对4组结构失稳时临界载荷的变化规律,分析表明,在定性分析上具有一致性,验证了利用有限元屈曲分析方法,计算单层E型瓦楞纸箱托盘临界载荷的可行性。由于研究对象是同一种材料,因此分析结果有一定的局限性。由于材料的差异,弹性模量、泊松比等参数需要经过试验测定,材料工程常数数据库有必要通过基础工作建立。

根据数据的分析,为了提高单层E型瓦楞纸箱托盘的结构稳定性,首先可以增加E型瓦楞纸板的刚度,其次可以优化E型瓦楞纸箱托盘的尺寸比例,然后可以增加E型瓦楞纸箱纸板层数来增加厚度等,以提高E型瓦楞纸箱托盘表面利用率,降低包装成本。

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